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SIGNED PRODUCT CORDIAL LABELING AND SIGNED TOTAL PRODUCT CORDIAL LABELING FOR SOME NEW GRAPHS

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ABSTRACT

A vertex labeling of graph G f: V(G) →{-1,1}with induced edge labeling f^*:E(G) → {-1,1} defined by $f^*(uv)$ = f(u)f(v) is called a signed product cordial labeling if $|v_f(-1) - v_f(1)| \leq 1$ and $|e_f(-1) - e_f(1)| \leq 1$ where v_f (-1) is the number of vertices labeled with '-1' and v_f (1) is the number of vertices labeled with '+1', e_f(-1) is the number of edges labeled with '-1' and e_f(1) is the number of edges labeled with '+1'. A graph G is signed product cordial if it admits signed product cordial labeling and Total signed product cordial labeling.

KEYWORDS: Labeling, Product and Total Product Cordial Labeling, Signed and Total Signed Product Cordial Labeling, Binary Tree

INTRODUCTION

Here I have begin with simple, finite and connected graph G= (V (G), E (G)) without loops and multiple edges.

I have discussed the definitions of labeling, cordial labeling, Product cordial labeling, Total Product cordial labeling, signed product cordial labeling and Total signed product cordial labeling. Also I have discussed few theorems in signed product cordial labeling. Finally, I have concluded the signed product cordial is Total signed product cordial.

Jayapal Baskar Babujee, Shobana Loganathan (2011),2, p.1525-1530 proved that path graph , cycle graphs, Star $k_{1,n}$, Bistar –$B_{n,n}$, P_n^+ , n≥3 and C_n^+ , n≥3 are signed product cordial.

There are two types of problems that are considered in this area,

- How the signed product cordiality is affected the various types of graph.
- Construct new families of signed product cordial graphs by finding suitable labeling.

A mapping f: V (G) → {0, 1} is called binary vertex labeling of G under f(v) is called label of vertex v of G under f. The induced edge labeling f^* : E(G) →{ 0, 1} is given by f^*(e=uv) = $|f(u) - f(v)|$.Let $V_f(0)$, $V_f(1)$ be the number of vertices labeled with 0 and 1 under f. Let $e_f(0)$, $e_f(1)$ be the number of edges of G having labels 0 and 1 respectively under f^*. A binary vertex labeling of graph G is called cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is cordial if it admits cordial labeling.

A vertex labeling of graph G f: V(G) → {-1,1} with induced edge labeling f^* : E(G)→{-1,1} defined by $f^*(uv)$ = f(u)f(v) is called signed product cordial labeling if $|v_f(-1) - v_f(1)| \leq 1$ and $|e_f(-1) - e_f(1)| \leq 1$ Where $V_f(-1)$

is the number of vertices labeled with -1, $V_f(1)$ is the number of vertices labeled with 1.$e_f(-1)$ is the number of edges labeled with -1 and $e_f(1)$ is the number of edges labeled with 1. A graph G is signed product cordial if it admits signed product cordial labeling.

Let f be a function from V(G) to { 0,1,2…….k-1} where k is an integer , $2 \leq k \leq |V(G)|$. For each edge uv, assign the label f(u)f(v) (mod k) f is called a k- Total product cordial labeling of G if $|f(i) - f(j)| \leq 1$ I,j ∈ {0,1,2……….k-1} where f(x) denotes the total number of vertices and edges labeled with x (x=0,1,2,………k-1). A graph with a k-total product cordial labeling is called a k-Total product cordial labeling.

Let f be a function from V(G) to {-1,1} with induced edge labeling $f^*: E(G) \rightarrow \{-1,1\}$ defined by $f^*(uv) = f(u)f(v)$ is called a signed product cordial if f is called a Total signed product cordial labeling if $|f(i) - f(j)| \leq 1$ where i,j ∈{-1,1} where f(x) denotes the total number of vertices and edges with x (x = -1,1). A graph with a k-Total signed product cordial labeling is called a k-Total signed product cordial graph.

MAIN RESULTS

Theorem 2.1

The flower graph F_n is k-signed product cordial where $2 \leq n \leq k$.

Proof

Let us consider the center vertex of the flower graph as u, labeling with '-1' . In that vertex u is called the hub vertex of the flower graph.

Let us denote the vertices in the cycle of the graph as u_1, u_2…….u_{n-1},u_n in the clockwise direction, labeling with '+1'. Denote the end vertices of the flower as v_1, v_2…………v_{n-1},v_n in the clockwise direction , labeling with '-1'.

Denote the edges incident with u as b_1,b_2……..b_{m-1},b_m in the clockwise direction, labeling with '-1'. Denote the edges of the cycle in the flower as e_1,e_2……..e_{m-1},e_m in the clockwise direction, labeling with'+1'. Denote the pendant edges of the flower graph as c_1, c_2, ………..c_{m-1},c_m in the clockwise direction, labeling with '-1'. Denote the edges which is attached to the center of the vertex u and with the pendant vertex as a_1, a_2, ………..a_{m-1},a_m again in the clockwise direction, labeling with '+1'.

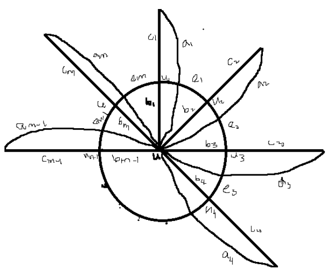

Figure 1

In order to compute the signed product cordial labeling

Let f be a function from V(G) to {-1,1} for each edge uv assign the label f(u)f(v) ie, uv=f(u)f(v).

Next we compute the signed product cordial labeling of the flower graph.

When n= 2, 3, 4, 5, 6, 7, 8…

The total number of vertices labeled with **-1** are given by $V_f(-1)$ = n+1 and the total number of vertices labeled with **1** are given by $V_f(1)$ = n.

Therefore, the total difference between the vertices labeled with **-1** and **1** are $|v_f(-1) - v_{f(1)}| = |(n+1) - n| = |1| = 1$

Similarly, the total number of edges labeled with **-1** and **1** are given by 2n. Therefore, the total difference between **-1** and **1** are given by $|e_f(-1) - e_{f(1)}| = |(2n) - 2n| = |0| = 0$.

Therefore $V_f(-1)$ = n+1, $V_f(1)$ = n and $e_f(-1) = e_f(1)$ =2n.

Hence the flower graph F_n , n≥ 2 admits signed product cordial labeling. Therefore the flower graph F_n is k-signed product cordial labeling.

Remarks

- The flower graph F_n is also satisfies the Total signed product cordial labeling.
- Tree is a connected graph and it has no circuits.
- An Ordered rooted tree is a binary tree if each vertex has at most two children. A full binary tree is a binary tree in which each internal vertex has exactly two children.

Theorem 2.5 Every Full Binary Tree is Signed Product Cordial Labeling

Proof

We know that every full binary tree has odd number of vertices and even number of edges.

Let T be a full binary tree and V be a root which is called zero level vertex. Clearly i^{th} level has 2^i vertices. If T has m levels then the number of vertices is 2^{m+1}-1 and the number of edges is 2^{m+1}-2.

Now assign the label **-1** to the root vertex V and assign labels **-1** and **+1** alternatively to the all level of the vertices.

For zero level a binary tree contains only one vertex that is called a root vertex marked as **'-1'**

Next we assign **-1** and **+1** to the first level of the vertices. Similarly we assign the labels up to **i^{th}** level of the vertices.

Hence $|v_f(-1) - v_f(1)| \leq 1$ and $|e_f(-1) - e_f(1)| \leq 1$. Every full binary tree is signed product cordial.

Remarks

Full binary tree is also satisfies Total signed product cordial.

K-SQUARE

Construction of Box fractal starts with a single square in the first iteration. Then the second iteration each side of the square is added with another square it leads to plus type five square .similarly for each next iteration all the external

square is added with one single square continue the same procedure until the **ith** iteration. It also leads with plus type until **ith** iteration.

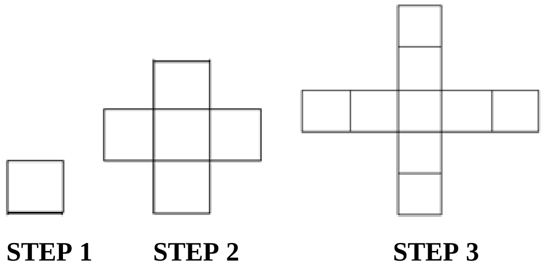

Figure 2

Theorem 3.1 Prove that k-Square is Signed and Total Signed Product Cordial

Proof

Let us start with Single Square in the first iteration. Then in the second iteration, each side of the square is added with another square , it leads to plus type five squares. Similarly, in the third iteration all the external square is added with Single Square it also leads to plus type nine squares. Continue the same pattern until the **ith** iteration.

The pattern of labeling the vertices are easily understandable by using the figure 3. Vertically all vertices in the column wise labeled with **+1** & **-1**alternatively. Hence in this type of labeling satisfies the signed product cordial labeling. Horizontally all the vertices in the (figure 3) below labeling pattern each row marked as **+1** & **-1** alternatively until the **ith** iteration. The pattern of labeling of edges is easily understandable by the below figure 3.

According to the condition of signed product cordial, labeling the edges in each iteration.

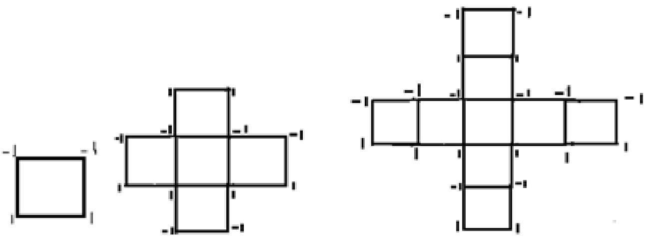

Figure 3

Initially a single square is labeled with two **+1** horizontally and two '**-1**' horizontally adjacent to each other and it satisfies the condition of signed product cordial labeling.

Then for the second iteration to satisfy condition for vertices and edges are labeled with **+1** and **-1** as given in the figure 3.

Hence for the third iteration also satisfy the condition for vertices and edges are labeled with **+1**'s and **-1**'s as given in the figure 3.

Continue the same pattern of labeling can be extended further. In each iteration it satisfies the conditions of signed product cordial labeling &total signed product cordial labeling.

Table 1

Iterations	Number of Squares	Vertices	Edges
1	S=1	$V_f(-1)=V_f(1)=2$	$e_f(-1)=e_f(1)=2$
2	S=5	$V_f(-1)=V_f(1)=6$	$e_f(-1)=e_f(1)=8$
3	S=9	$V_f(-1)=V_f(1)=10$	$e_f(-1)=e_f(1)=14$
4	S=13	$V_f(-1)=V_f(1)=14$	$e_f(-1)=e_f(1)=20$
.	.	.	.
.	.	.	.
.	.	.	.
N	$S_n= S_{n-1}+4$	$v_{f(n)} = v_{f(n-1)}+4$	$e_{f(n)} =e_{f(n-1)}+6$

Theorem 3.2 Prove that Signed Product Cordial Labeling is a Total Signed Product Cordial Labeling

Proof

Normally every simple &connected graph by it satisfies the condition of signed product cordial labeling in this case $v_{f(-1)}$ & $v_{f(1)}$moreover it should be equal and also $e_{f(-1)}$ &$e_{f(1)}$ should be equal. According to the definition of Total signed product cordial labeling $|v_f(-1) + e_f(-1) - v_f(1) + e_f(1)| \leq 1$. Therefore every signed product cordial labeling is a Total signed product cordial labeling.

Remarks

- Every signed product cordial labeling is a Total signed product cordial labeling.
- In signed product cordial labeling moreover $v_f(-1)$ &$v_f(1)$ are equal and $e_f(-1)$ &$e_f(1)$ are equal.

CONCLUSIONS

Every graph admit signed Product cordial labeling it is very interesting to investigate the graph which admit signed product cordial labeling. In this paper I have introduced the concept of Total signed product cordial labeling. Also investigate some new graphs of signed and Total signed product cordial labeling. To derive similar results on other graph families in open area of research.

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